Operational Semantics of μScheme

April 10, 2024

Logistics

- Assignment #3 will be out soon

ImpCore evaluation

$$\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle \qquad \langle d, \xi, \phi \rangle \rightarrow \langle \xi', \phi' \rangle$$

ImpCore evaluation

$$\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle \qquad \langle d, \xi, \phi \rangle \rightarrow \langle \xi', \phi' \rangle$$

- e: expression to evaluate
- ξ: global environment
- ϕ: function environment (does not change in evaluation!)
- ρ: parameter environment

μScheme evaluation

$$\langle e, \rho, \sigma \rangle \Downarrow \langle v, \sigma' \rangle \qquad \langle d, \rho, \sigma \rangle \rightarrow \langle \rho', \sigma' \rangle$$

μScheme evaluation

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- ρ maps variables to locations l

μScheme evaluation

$$\langle e, \rho, \sigma \rangle \Downarrow \langle v, \sigma' \rangle \qquad \langle d, \rho, \sigma \rangle \to \langle \rho', \sigma' \rangle$$

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- $l \in \{l_0, l_1, l_2, \ldots\}$ represents an address in memory

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- $l \in \{l_0, l_1, l_2, \ldots\}$ represents an address in memory
- σ is the memory: it maps locations to values.

μScheme evaluation

$$\langle e, \rho, \sigma \rangle \Downarrow \langle v, \sigma' \rangle \qquad \langle d, \rho, \sigma \rangle \rightarrow \langle \rho', \sigma' \rangle$$

- ρ maps variables to locations l (does not change in evaluation)
- $l \in \{l_0, l_1, l_2, \ldots\}$ represents an address in memory
- σ is the memory: it maps locations to values.
- The memory *can* change (though we haven't used this)

Main challenge: managing state of closures

Variables

$$\frac{x \in \mathrm{dom}(\rho) \qquad \rho(x) \in \mathrm{dom}(\sigma)}{\langle \mathsf{VAR}(x), \rho, \sigma \rangle \Downarrow \langle \sigma(\rho(x)), \sigma \rangle}$$

$$\frac{x \in \mathrm{dom}(\rho) \qquad \rho(x) = l \qquad \langle e, \rho, \sigma \rangle \Downarrow \langle v, \sigma' \rangle}{\langle \mathsf{SET}(x, e), \rho, \sigma \rangle \Downarrow \langle v, \sigma'\{l \mapsto v\} \rangle}$$

Local variables

μScheme allows us to declare local variables

```
(let ((x value-of-x)
      (y value-of-y)
      ...)
  expression-with-x-and-y)
```

Case of one variable declaration:

$$\frac{l_x \notin \mathrm{dom}(\sigma_1) \qquad \langle e_x, \rho, \sigma_0 \rangle \Downarrow \langle v_x, \sigma_1 \rangle \quad \langle e, \rho\{x \mapsto l_x\}, \sigma_1\{l_x \mapsto v_x\} \rangle \Downarrow \langle v, \sigma_2 \rangle}{\langle \mathsf{LET}(x, e_x, e), \rho, \sigma_0 \rangle \Downarrow \langle v, \sigma_2 \rangle}$$

Functions

$$\frac{}{\langle \texttt{LAMBDA}(x, e), \rho, \sigma \rangle \Downarrow \langle (\!| \texttt{LAMBDA}(x, e), \rho |\!), \sigma \rangle}$$

$$\frac{}{\langle \texttt{LAMBDA}(x, e), \rho, \sigma \rangle \Downarrow \langle (\!| \texttt{LAMBDA}(x, e), \rho |\!), \sigma \rangle}$$

$$\frac{l_x \notin \mathrm{dom}(\sigma) \qquad \langle e, \rho, \sigma \rangle \Downarrow \langle (\!| \texttt{LAMBDA}(x, e_c), \rho_c |\!), \sigma_0 \rangle \\ \langle e_x, \rho, \sigma_0 \rangle \Downarrow \langle v_x, \sigma_1 \rangle \qquad \langle e_c, \rho_c\{x \mapsto l_x\}, \sigma_1\{l_x \mapsto v_x\} \rangle \Downarrow \langle v, \sigma' \rangle}{\langle \texttt{APPLY}(e, e_x), \rho, \sigma \rangle \Downarrow \langle v, \sigma' \rangle}$$

Supporting multiple arguments/variables

- Check that variables are distinct
- Create multiple fresh locations
- Evaluate arguments/initial values left to right

Example

Evaluate the following expression

```
(let ((f (let ((x 1))
           (lambda (y) (set x (+ x y))))))
  (f (f 5)))
```

First, evaluate the lambda

Let's abbreviate

$$b_f = \mathtt{SET}(x, (\mathtt{APPLY}(\mathtt{VAR}(+), \mathtt{VAR}(x), \mathtt{VAR}(y))))$$
$$e_f = \mathtt{LAMBDA}(y, b_f)$$
$$c_f = (\!| e_f, \{x \mapsto l_x\} |\!)$$

Derive

$$D_\lambda = \frac{}{\langle e_f, \{x \mapsto l_x\}, \{l_x \mapsto 1\}\rangle \Downarrow \langle c_f, \{l_x \mapsto 1\}\rangle}$$

Next, evaluate the inner let

$$D_x = \frac{\overline{\langle 1, \emptyset, \emptyset \rangle \Downarrow \langle 1, \emptyset \rangle} \qquad l_x \notin \mathrm{dom}(\emptyset) \qquad D_\lambda}{\langle \mathsf{LET}(x, 1, e_f), \emptyset, \emptyset \rangle \Downarrow \langle c_f, \{l_x \mapsto 1\} \rangle}$$

Evaluate the first call

$$\rho_0 = \{f \mapsto c_f\}$$
$$\rho_1 = \{x \mapsto l_x, y \mapsto l_y\}$$
$$\sigma_0 = \{l_f \mapsto c_f, l_x \mapsto 1\}$$
$$\sigma_1 = \{l_f \mapsto c_f, l_x \mapsto 1, l_y \mapsto 5\}$$
$$\sigma_2 = \{l_f \mapsto c_f, l_x \mapsto 6, l_y \mapsto 5\}$$
$$l_y \notin \{l_f, l_x\}$$

$$D_{\mathtt{f5}} = \frac{\dfrac{\cdots}{\langle \mathrm{VAR}(f), \rho_0, \sigma_0\rangle \Downarrow \langle c_f, \sigma_0\rangle} \qquad \dfrac{}{\langle 5, \rho_0, \sigma_0\rangle \Downarrow \langle 5, \sigma_0\rangle} \qquad \dfrac{\cdots}{\langle b_f, \rho_1, \sigma_1\rangle \Downarrow \langle 6, \sigma_2\rangle}}{\langle \mathrm{APPLY}(\mathrm{VAR}(f), 5), \rho_0, \sigma_0\rangle \Downarrow \langle 6, \sigma_2\rangle}$$

Evaluate the second call

$$\rho_2 = \{x \mapsto l_x, y \mapsto l'_y\}$$
$$\sigma_3 = \sigma_2\{l'_y \mapsto 6\}$$
$$\sigma_4 = \sigma_3\{l_x \mapsto 12\}$$

$$D_{\texttt{f(f5)}} = \dfrac{l'_y \notin \{l_f, l_x, l_y\} \qquad \dfrac{\cdots}{\langle \mathsf{VAR}(f), \rho_0, \sigma_0\rangle \Downarrow \langle c_f, \sigma_0\rangle} \qquad D_{\texttt{f5}} \qquad \dfrac{\cdots}{\langle b_f, \rho_2, \sigma_3\rangle \Downarrow \langle 12, \sigma_4\rangle}}{\langle \mathsf{APPLY}(\mathsf{VAR}(f), \mathsf{APPLY}(\mathsf{VAR}(f), 5)), \rho_0, \sigma_0\rangle \Downarrow \langle 12, \sigma_4\rangle}$$

Everything combined

$$D_{\mathtt{f}} = \frac{l_f \notin \emptyset \qquad D_{\mathtt{x}} \qquad D_{\mathtt{f(f5)}}}{\langle \mathsf{LET}(f, \mathsf{LET}(x, 1, e_f), \mathsf{APPLY}(\mathsf{VAR}(f), \mathsf{APPLY}(\mathsf{VAR}(f), 5))), \emptyset, \emptyset\rangle \Downarrow \langle 12, \{l_f \mapsto c_f, l_x \mapsto 12, l_y \mapsto 5, l'_y \mapsto 6\}\rangle}$$